

An Open Quantum System Approach to the B-Mesons System

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In this paper we consider a nonstandard evolution for the $B^0-\bar{B}^0$ system, namely, an evolution in the open quantum systems framework. Such approach is justified by the very high sensitivity of experiments studying CP-violating phenomena in the B-mesons sector, very near to the one required to test some possible scenarios induced by quantum gravity at the Planck scale, whose effects at low energy can be described by a heat bath. We adopt a phenomenological point of view, introducing 6 new parameters that fully describe this kind of evolution without referring to a specific model for the microscopic interaction. We outline the main differences between this approach and the usual one in the description of evolution and decay of single mesons or correlated pairs.

KEY WORDS: Quantum open systems; Irreversible Dynamics; complete positivity; B-mesons physics.

1. INTRODUCTION

The physics of B-flavored hadrons is at present one of the most promising fields testing Standard Model predictions concerning some interesting physical processes like CP-violating phenomena and flavour oscillations. Presently, several experiments are collecting data; other very accurate experiments are planned to start in the near future. In this context it is also possible to study some exotic scenarios of physics beyond the Standard Model; in this contribution we consider an open quantum system treatment for the $B^0-\bar{B}^0$ system (Benatti *et al.*, 2001) producing decoherence.²

A wide variety of physical systems has been studied in this framework. The formalism of open quantum systems, initially developed in quantum optics (Gardiner and Zoller, 2000; Louisell, 1973; Scully and Zubairy, 1997), has also been applied to study phenomena of quantum relaxation in magnetic resonance (Slichter, 1990), to foundational aspects of quantum mechanics (Fonda *et al.*, 1978)

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² For a preliminary analysis see Benatti and Floreanini (1999a); for a different approach to decoherence in the B-mesons system see Bertlmann and Grimus (2001).

and, lately, to model some typical situations in particle physics: neutral mesons systems (Ellis *et al.*, 1996; Huet and Peskin, 1995), neutrinos (Benatti and Floreanini, 2000a, 2001), photons (Benatti and Floreanini, 2000b), neutrons interferometry (Benatti and Floreanini, 1999d).

An open system (Alicki and Lendi, 1987; Gorini *et al.*, 1978; Spohn, 1980) is a system S that interacts with its external environment E , and, then, exchanging energy and entropy with it. Although the dynamics of the composite system $T = S + E$ is unitary, this is no longer true for the dynamics of the system S alone, obtained tracing the time-evolution of T over the degrees of freedom of the environment E . In general, the resulting reduced dynamics is complicated by memory effects. It is possible to get a Markovian evolution for the system S (i.e., a memoryless dynamics) if essentially two assumptions hold: the interaction between S and E must be weak, and there must be no initial correlation between them (Alicki and Lendi, 1987; Gorini *et al.*, 1978; Spohn, 1980).

Since we are interested in time-evolutions causing decoherence, that is transitions from pure states to mixtures, we must describe the state of S by means of statistical operators ρ , that is Hermitian and positive trace-class matrices with unit trace. The time-evolution for ρ is given by

$$\dot{\rho}(t) = L[\rho(t)] = -iH\rho(t) + i\rho(t)H^\dagger + L_D[\rho(t)], \quad (1)$$

where L is the generator of the dynamics, H is the Weisskopf–Wigner Hamiltonian describing the system S (non-Hermitian because the system is unstable) and L_D is the generator of the dissipative part, embodying the interaction with the environment. The form of L_D , univocally fixed by the Kossakowski-Lindblad theorem (Alicki and Lendi, 1987; Gorini *et al.*, 1978), is very general since it characterizes any Markovian dynamics:

$$L_D[\rho(t)] = -\frac{1}{2} \left\{ \sum_i A_i^\dagger A_i, \rho(t) \right\} + \sum_i A_i \rho(t) A_i^\dagger, \quad (2)$$

where the set of operators A_i , depends on the details of the interaction.

The time-evolution maps $\rho(0) \rightarrow \rho(t) = \Lambda_t[\rho(0)]$, where $\Lambda_t = e^{Lt}$ satisfy the forward in time composition law: $\Lambda_{t+s} = \Lambda_t \circ \Lambda_s$ with $t, s \geq 0$ characteristic of irreversible phenomena; furthermore, they are completely positive (Alicki and Lendi, 1987; Gorini *et al.*, 1978). This last property is noteworthy in particle physics, because it guarantees the preservation of positivity of evolving density matrices describing entangled systems. Indeed, simple positivity is not sufficient for a proper statistical interpretation of the formalism of density matrices when they represent entangled systems (Alicki and Lendi, 1987; Gorini *et al.*, 1978) (counter examples are found in (Benatti and Floreanini, 1997a,b, 1998, 1999a,b,c)). Moreover, the evolution maps Λ_t allow transitions from pure to mixed states.

The entropy of open systems can increase as well as decrease; nevertheless, in what follows, we will consider only entropy increasing time-evolutions because we are interested in reduced dynamics producing states less ordered as time passes.

As we shall shortly see in the next section, the phenomenological approach to the $B^0-\bar{B}^0$ system is usually performed in the two-dimensional Hilbert space of flavors; then we shall consider 2×2 density matrices ρ that can be expanded over the Pauli matrices $\rho_i (i = 1, 2, 3)$ and the 2×2 identity ρ_0 :

$$\rho = \sum_{i=0}^3 \rho_i \sigma_i. \tag{3}$$

It is possible to give a vectorial representation of the density matrix ρ , namely, defining a 4-vector $|\rho\rangle$ whose components are the real coefficients ρ_i , in (3):

$$|\rho\rangle \equiv \begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix},$$

accordingly, since Eq. (1) is linear in time, it can be rewritten as a Schrödinger-like equation:

$$\partial_t |\rho(t)\rangle = (\mathcal{H} + \mathcal{D}) |\rho(t)\rangle, \tag{4}$$

where \mathcal{H} and \mathcal{D} are 4×4 real matrices, the Hamiltonian and the dissipative part, respectively. The nonstandard part of the dynamics is embodied in the symmetric matrix \mathcal{D} :

$$\mathcal{D} = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \alpha & \beta \\ 0 & c & \beta & \gamma \end{pmatrix}, \tag{5}$$

with $a, b, c, \alpha, \beta, \gamma$ six real phenomenological constants, constrained by the inequalities, necessary and sufficient to Complete Positivity (Alicki and Lendi, 1987):

$$\begin{aligned} 2R &\equiv \alpha + \gamma - a \geq 0, & RS &\geq b^2, \\ 2S &\equiv \alpha + \gamma - \alpha \geq 0, & RT &\geq c^2, \\ 2T &\equiv a + \alpha - \gamma \geq 0, & ST &\geq \beta^2, \\ RST &\geq 2bc\beta + R\beta^2 + Sc^2 + Tb^2. \end{aligned} \tag{6}$$

Since these coefficients are small³ with respect to the entries of \mathcal{H} , it is possible to solve (4) up to first order in these parameters:

$$|\rho(t)\rangle = e^{\mathcal{H}t} |\rho(0)\rangle + \int_0^t ds e^{\mathcal{H}(t-s)} \mathcal{D} e^{\mathcal{H}s} |\rho(0)\rangle. \tag{7}$$

³ A rough evaluation of their magnitude can be done: $D_{ij} \sim \epsilon_S^2 / \epsilon_E$ where ϵ_S and ϵ_E are the characteristic energies of the system and of the environment, respectively.

The first term describes the standard evolution of the density matrix, the second one accounts for the interaction with the environment.

In the following we won't report the explicit form of statistical operators at time t because it is not physically relevant. The computations have been performed in vectorial representation and using the approximation (7).

2. DECAY OF NEUTRAL B-MESONS

The neutral B-mesons system can be represented by statistical operators ρ acting on the two-dimensional flavour Hilbert space, spanned by the physical states $|B^0\rangle$ and $|\bar{B}^0\rangle$. In a dissipative context, their time-evolution in vectorial representation is given by (7); the computation has been performed working on the basis of the eigenstates of the Hamiltonian H , denoted by $|B_L\rangle$ and $|B_H\rangle$, where the subscripts mean "light" and "heavy," respectively:

$$\begin{cases} H|B_H\rangle = \lambda_H|B_H\rangle \\ H|B_L\rangle = \lambda_L|B_H\rangle \end{cases}, \quad \lambda_{H,L} = m_{H,L} - \frac{i}{2}\gamma_{H,L}, \tag{8}$$

where $m_{H,L}$ and $\gamma_{H,L}$ are the masses, respectively the widths, of the corresponding eigenstates.

This choice of basis provides an invariant formalism under independent-phase redefinitions of the two basis vectors. Therefore, it makes no commitment on the magnitude of CP (and CPT) violating effects.

In the density matrix formalism, the mean value of an observable \mathcal{O} is obtained from

$$\langle \mathcal{O}(t) \rangle = \text{Tr}[\mathcal{O}\rho(t)]. \tag{9}$$

Then, the probability rates for the decays of B^0 and \bar{B}^0 in different channels, $B^0, \bar{B}^0 \rightarrow f$, can be written as

$$\begin{aligned} \mathcal{P}_f(B^0, t) &= \text{Tr}[\mathcal{O}_f\rho(t)], \quad \text{with } \rho(0) = |B^0\rangle\langle B^0|, \\ \mathcal{P}_f(\bar{B}^0, t) &= \text{Tr}[\mathcal{O}_f\rho(t)], \quad \text{with } \rho(0) = |\bar{B}^0\rangle\langle \bar{B}^0|, \end{aligned}$$

where the operator \mathcal{O}_f is defined by

$$\mathcal{O}_f = \begin{pmatrix} o_1 & o_3 \\ o_4 & o_2 \end{pmatrix}, \tag{10}$$

with entries

$$\begin{aligned} o_1 &= |\mathcal{A}(B^0 \rightarrow f)|^2, & o_3 &= [\mathcal{A}(B^0 \rightarrow f)]^* \mathcal{A}(\bar{B}^0 \rightarrow f), \\ o_2 &= |\mathcal{A}(\bar{B}^0 \rightarrow f)|^2, & o_4 &= [\mathcal{A}(\bar{B}^0 \rightarrow f)]^* \mathcal{A}(B^0 \rightarrow f), \end{aligned}$$

where \mathcal{A} the the decay amplitude for the considered process.

The standard dynamics is characterized by $\Delta m = m_H - m_L$, $\Delta\Gamma = \gamma_L - \gamma_H$, and $\Gamma = \gamma_L + \gamma_H/2$; it is customary to renormalize these parameters, and the time variable, in the following way:

$$\omega = \frac{\Delta m}{\Gamma}, \quad \delta = \frac{\Delta\Gamma}{2\Gamma}, \quad \tau = t\Gamma. \quad (11)$$

The dissipative contribution is instead parameterized by particular functions of a, b, c, α, β , and γ , denoted by A, B, C , and D ; their expressions are reported in Benatti *et al.* (2001).

2.1. Decay of a Single B-Meson

Typical decays relevant for the study of CP (and CPT) violation, are the so-called semileptonic decays, in which the final state f contains a hadron h , a lepton l , and a neutrino (or antineutrino): $B^0(\bar{B}^0) \rightarrow h^-l^+\nu(h^+l^-\bar{\nu})$.

It is customary to parameterize the amplitudes in (10) as follows (Bigi and Silva, 2000; Branco *et al.*, 1999):

$$\begin{aligned} \mathcal{A}(B^0 \rightarrow h^-l^+\nu) &= \mathcal{M}_h(1 - y_h), \\ \mathcal{A}(\bar{B}^0 \rightarrow h^+l^-\bar{\nu}) &= \mathcal{M}_h^*(1 + y_h^*), \\ \mathcal{A}(B^0 \rightarrow h^+l^-\bar{\nu}) &= z_h \mathcal{A}(\bar{B}^0 \rightarrow h^+l^-\bar{\nu}), \\ \mathcal{A}(\bar{B}^0 \rightarrow h^-l^+\nu) &= x_h \mathcal{A}(B^0 \rightarrow h^-l^+\nu), \end{aligned}$$

where the parameters x_h and z_h measure violations of the rule $\Delta B = \Delta Q$, satisfied in the Standard Model, and the coefficient y_h measures deviations from CPT invariance.

For sake of simplicity, the results we show below are obtained assuming $x_h = z_h = y_h = 0$: more complete results are collected in Benatti *et al.* (2001). The probability rates present new dissipative terms absent in the standard case; for example, in the case of the decay $B^0 \rightarrow h^+l^-\bar{\nu}$:

$$\begin{aligned} \mathcal{P}_{h^+}(B^0, \tau) &= \frac{|\mathcal{M}_h|^2}{2} e^{-t} \left\{ \cosh \delta \cdot \tau - \cos \omega \tau e^{-(A-D)\tau} \right. \\ &\quad \left. + \sinh \delta \tau \left(\frac{D}{\delta} - \frac{4\omega}{\delta^2 + \omega^2} \text{Im}(C) \right) \right. \\ &\quad \left. + \sin \omega \tau \left(\frac{4\omega}{\delta^2 + \omega^2} \text{Im}(C) - \text{Re}(B) \right) \right\} \quad (12) \end{aligned}$$

If dissipation is neglected ($A = B = C = D = 0$) we obtain the standard expression of the probability rate: the second row of (12) vanishes and the damping exponential in the first row disappears.

From a fit of the time-dependence of (12) it should be possible, at least in principle, to get the parameters A , D , $\text{Re}(B)$, and $\text{Im}(C)$.

Usually, instead of analyzing directly the decay rates, some particular asymmetries, constructed by means of these rates, are taken into account. The following one, for example, is extensively studied:

$$\mathcal{A}_{\Delta m}(\tau) \equiv \frac{[\mathcal{P}_{h^-}(B^0, \tau) - \mathcal{P}_{h^-}(\bar{B}^0, \tau)] - [\mathcal{P}_{h^+}(B^0, \tau) - \mathcal{P}_{h^+}(\bar{B}^0, \tau)]}{\mathcal{P}_{h^-}(B^0, \tau) + \mathcal{P}_{h^-}(\bar{B}^0, \tau) + \mathcal{P}_{h^+}(B^0, \tau) + \mathcal{P}_{h^+}(\bar{B}^0, \tau)}. \quad (13)$$

Indeed, in the standard case, this quantity is used to fit the parameter Δm introduced previously; when we consider the nonstandard evolution it assumes the simple form:

$$\mathcal{A}_{\Delta m}(\tau) = e^{-A\tau} \cos \omega\tau + \text{Re}(B) \sin \omega\tau \quad (14)$$

that reduces, in the standard case (i.e., $A = B = C = D = 0$) to $\mathcal{A}_{\Delta m}(\tau) = \cos \omega\tau$. We see that a study of the time-dependence of this quantity should provide, in principle, the dissipative parameters A and $\text{Re}(B)$.

The evaluation of the parameter Δm in the standard case is also performed considering time-integrated rates

$$\mathcal{P}_f(B) = \frac{1}{\Gamma} \int_0^\infty d\tau \mathcal{P}_f(B, \tau) \quad (15)$$

and introducing the asymmetry $\mathcal{A}'_{\Delta m}$, defined in analogy to (13), with the time-integrated rates; the expression of this quantity in the dissipative context is

$$\mathcal{A}'_{\Delta m} = \frac{1}{1 + \omega^2} \left\{ 1 + \omega \text{Re}(B) + \frac{1}{1 + \omega^2} [(\omega^2 - 1)A - 2\omega^2 D] \right\}. \quad (16)$$

In this case, since the dependence on time is lost, it is more difficult to constrain the parameters A , $\text{Re}(B)$, and D . This is a general problem for time-integrated quantities.

Another class of relevant decays of single B-mesons has a final state that is a CP eigenstate, for example: $f = D^+ D^-$, $\pi^+ \pi^-$, $J/\psi K$. In this case it is convenient to define a new asymmetry:

$$\mathcal{A}_f(t) \equiv \frac{\mathcal{P}_f(B^0, \tau) - \mathcal{P}_f(\bar{B}^0, \tau)}{\mathcal{P}_f(B^0, \tau) + \mathcal{P}_f(\bar{B}^0, \tau)}. \quad (17)$$

This quantity would be rigorously zero in absence of CP violation; it is studied because it enables to fit the parameter β appearing in the CKM matrix (Bigi and Silva, 2000; Branco *et al.*, 1999). With the nonstandard contributions to the

time-evolution it takes the form:

$$\begin{aligned} \mathcal{A}_f(\tau) = & \frac{2\xi_f}{\omega} \text{Im}(C) - \sin \omega\tau \left(\frac{2 \text{Im}(\lambda_f)}{1 + |\lambda_f|^2} + \frac{2\xi_f}{\omega} \text{Re}(C) \right) \\ & + \cos \omega\tau \left(\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} - \frac{2\xi_f}{\omega} \text{Im}(C) \right), \end{aligned} \quad (18)$$

where λ_f is a phase-independent coefficient introduced in order to parameterize the neutral mesons decays (Bigi and Silva, 2000; Branco *et al.*, 1999). A fit of this asymmetry would provide information about the parameter C .

2.2. Decay of Correlated B-Mesons

Of particular interest are the decays of pairs of entangled B-mesons, produced at the so-called B-factories. As already stated, the property of Complete Positivity plays a fundamental role in this case.

The entangled B-mesons are generated from the decay of the $\Upsilon(4S)$ resonance: $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$; the initial state is $|\psi_B\rangle = \frac{1}{\sqrt{2}}(|B^0, -p\rangle \otimes |\bar{B}^0, p\rangle - |\bar{B}^0, -p\rangle \otimes |B^0, p\rangle)$ and the initial density matrix $\rho(0) = |\psi_B\rangle\langle\psi_B|$. Its time-evolution is supposed to be factorizable:

$$\rho(t_1, t_2) = (\Lambda_{t_1} \otimes \Lambda_{t_2})\rho(0), \quad (19)$$

where the first meson has evolved up to a time t_1 and the second up to t_2 . The probability of decay for the first meson in a final state f_1 at time t_1 and for the second meson in f_2 at t_2 is given by

$$\mathcal{P}(f_1, t_1; f_2, t_2) = \text{Tr}[(\mathcal{O}_{f_1} \otimes \mathcal{O}_{f_2})\rho(t_1, t_2)], \quad (20)$$

where the operators \mathcal{O}_f have been introduced in (10).

In the experimental analysis single time distributions are usually taken into account; writing $t_1 = t' + t$ and $t_2 = t'$ we express these probabilities by

$$\mathcal{P}_{f_1, f_2}(t) = \int_0^{+\infty} dt' \mathcal{P}(f_1, t' + t; f_2, t') \quad (21)$$

for positive t , i.e., when the first meson decays after the second one, and by

$$\mathcal{P}_{f_1, f_2}(-|t|) = \int_0^{+\infty} dt' \mathcal{P}(f_1, t' + t; f_2, t') \theta(t' - |t|) \quad (22)$$

for negative t (when the first meson decays before). These definitions fulfill the logical request: $\mathcal{P}_{f_1, f_2}(-|t|) = \mathcal{P}_{f_2, f_1}(|t|)$.

We have analyzed the behavior of some characteristic quantities of the double semileptonic decays, i.e., those decays in which both mesons undergo a

semileptonic decay. The first quantity we consider is the asymmetry:

$$\mathcal{R}(t) \equiv \frac{\mathcal{P}_{h^+,h^+}(t) + \mathcal{P}_{h^-,h^-(t)}}{\mathcal{P}_{h^+,h^-(t)} + \mathcal{P}_{h^-,h^+(t)}}. \quad (23)$$

Its relevant property is that in the standard case $\mathcal{R}(0) = 0$, because $\mathcal{P}_{f,f}(0) = 0$, whereas taking in account the dissipative contributions it becomes

$$\mathcal{R}(0) = \frac{1}{2} \left[A + \frac{\omega}{1 + \omega^2} (\omega \operatorname{Im}(B) - \operatorname{Re}(B)) \right], \quad (24)$$

regardless of the assumptions made about the coefficients x_h , z_h , and y_h . Then a nonzero value of $\mathcal{R}(0)$ clearly shows the presence of dissipative phenomena.

Another interesting asymmetry is expressed by

$$\mathcal{A}_{\Delta m}(\tau) \equiv \frac{[\mathcal{P}_{h^+,h^-(\tau)} + \mathcal{P}_{h^-,h^+(\tau)}] - [\mathcal{P}_{h^+,h^+(\tau)} + \mathcal{P}_{h^-,h^-(\tau)}]}{\mathcal{P}_{h^+,h^+(\tau)} + \mathcal{P}_{h^-,h^-(\tau)} + \mathcal{P}_{h^+,h^-(\tau)} + \mathcal{P}_{h^-,h^+(\tau)}}. \quad (25)$$

It is analogous to the one defined in (13) for the single meson system, because it allows, if there is not dissipation, the determination of the parameter Δm . In our case the dependence on the dissipative parameters is

$$\mathcal{A}_{\Delta m}(\tau) = \frac{e^{-A\tau}}{1 + A} \cos \omega\tau + (\omega \cos \omega\tau + \sin \omega\tau) \operatorname{Re} \left(\frac{B}{1 - i\omega} \right). \quad (26)$$

In the above expression we have assumed $\delta = 0$, a good approximation since $\Delta\Gamma \ll \Gamma$. From the time-dependence of the contributions in (26) it should be possible to evaluate the dissipative parameters A and B .

3. CONCLUSIONS

We have treated the neutral B-mesons system in the framework of quantum dynamical semigroups. The formalism is very general and can be applied to describe the time evolution of this system when it is subject to a weak interaction with an external environment. For instance, quantum gravity could give a motivation for such an approach, since in this case the space-time itself would act as an effective environment for any physical system.

The open system framework allows a rough evaluation of magnitude of the dissipative parameters; they scale at most as m_B^2/M_F , where m_B is the meson mass whereas M_F is a large fundamental mass scale. Typically, M_F coincides with the Planck scale so that $m_B^2/M_F \sim 10^{-18}$ GeV. This value is very small; however, the sophistication of the dedicated B-experiments, both at colliders (CDF-II, HERA-B, BTeV, LHC-*b*) and B-factories (BaBar, Belle, CLEO-III), is so high that the sensitivity required to probe the presence of nonvanishing dissipative parameters can be reached in just a few years of data taking (BaBar Collaboration, 1998; Ball *et al.*, 2000; BTeV Proposal, 2000). If these parameters should be found not

different from zero, the inequalities expressing the Complete Positivity could be tested. Then, the neutral B-mesons physics is potentially a good laboratory to test basic properties of open systems dynamics.

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